Probabilistic Framework for Evaluation of Smart Grid Resilience of Cascade Failure

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***Abstract*— The next generation power grid demands high reliability, robustness and real time communication of control information related to power flow in the grid. This paper proposes a probabilistic framework of smart grid power network with statistical decision theory to evaluate system performance in steady state as well as under dynamical case and identify the probable critical links which can cause cascade failure. Proposed model for cascade failure prediction has been tested on the IEEE 30 bus test bed system. Simulation results validated critical links in probabilistic model of power grid system with deterministic power flow analysis. The key contribution of this paper is, performance evaluation of smart grid power network and identification as well as prediction of critical links which may lead to system blackout. In addition to this, a graphical model has been developed using minimum spanning tree to analyze topology and structural connectivity of IEEE 30 bus system.**

Keywords: Cumulative distribution function, Minimum spanning tree, Phasor Measurement Unit, Power grid topology, Probability density function.

I. INTRODUCTION

The smart grid is an integration of communication infrastructure over existing power grid infrastructure with real time communication of control information like normal grid operation or information related to any abnormality/line tripping in the grid. A review on bidirectional communication/power flow was given in [1]. The Data control management unit interface communication and power network via Phasor Measurement Unit (PMU).

A PMU in a smart grid collects online as well as offline data from wide area network and synchronizes voltage and current measurement with a common time reference. PMU measurements are converted to phasors at different nodes with same time space coordinates and use these information for state estimation, error detection, control and outage monitoring of smart grid. A detailed survey of PMU placement technologies in smart grid were explained in [2], [3], [4], [5], [6] thorough description of the optimization methods. The load flow data from PMU need to be modeled and analysed via deterministic and probabilistic load flow models. Information from load flow models helps further in analysis and prediction of cascade link failure. A probabilistic load flow model was proposed for stability of power system by [7] in terms of density function. The paper compares the results obtained probabistically with those that would be obtained deterministically. DC power flow problem, using sensitivity coefficient of each line was probabilistically analysed in [8]. Simulation results indicate that, the changes in nodal data influence the power flow in the transmission line.

A probability theory used by [9], to solve load flow in power grid. The method applies two Maximum Entropy methods and a Gram-Charlier expansion to generate voltage magnitude, voltage angle and power flow probability density functions based on cumulant arithmetic treatment of linearised power flow equations. A model of cascading failures in interdependent network systems was developed in [10] and simulation results showed that at critical load the average cascade size give essential information on the system vulnerability towards cascading failures. The impact of network uncertainties in power systems were modeled in [11], [12] using distribution factor concept with the Monte Carlo simulations for computation of probabilistic load flow. Probabilistic load flow method considering random branch outages as well as uncertainties of nodal power injections were simulated in [13] to solve the discrete distribution part of each state as well as output variable after contingency.

This paper is an effort towards modeling and simulation of power grid for performance evaluation of load flow under steady state (normal)case and dynamic case (line tripping) with less computational complexity. Proposed probabilistic framework predict critical links which can lead to blackout and inform data control management unit of smart grid for further action. The paper is organized as follows: Section II presents deterministic and probabilistic load flow model. The statistical decision theory is explained in Section III.
Section IV gives graphical modeling and perfor-

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mance evaluation of IEEE 30 bus test bed system in normal grid operation and under cascading of line failure with simulation results conclusions are presented in Section V along with future directions for research.

II. POWER FLOW MODEL

The purpose of power flow model is to study and analyze the performance of load flow both in normal operating conditions and under fault (line tripping) condition.

A. Deterministic load flow model

Power grid is a complex network where generator and load buses are represented as a node and transmission line, transformers as a link. If P and Q represent real and reactive power of the system [14] then real and reactive powers injected into the i^{th} bus (node) are

$$P_i(Real\ power) = Re(V_i^* \sum_{k=1}^{n} Y_{ik} V_k) \quad (1)$$

$$Q_i(Reactive\ power) = -Im(V_i^* \sum_{k=1}^{n} Y_{ik} V_k) \quad (2)$$

where, Y_{ik}= line node admittance matrix
V_i = Voltage at the i^{th} node
V_k = Voltage across the k^{th} line

AC load flow equations are non-linear and complex hence, computational complexity is more with AC load flow analysis compare to DC. In the DC load flow model the net power injected into a node is real and equal to the total amount of power flowing to neighboring nodes through links (transmission lines or transformers) is

$$P_i = \frac{j(\theta_i - \theta_j)}{X_{ij}} \quad (3)$$

where, θ_i , θ_j is the voltage phase angle at node i and j and X_{ij} is the series reactance of the link between nodes i and j. Deterministic load flow models finds line flow under a specific operating condition and good for load flow analysis in steady state but analysis will be difficult in dynamical case because of the load variation in the system.

B. Probabilistic load flow model

The load variations in power grid like node voltage, current and power flow in transmission lines are treated as random variables in probabilistic model with normal distribution, hence, mean, variance, covariance and correlations for the nodal data can be easily calculated. Hence, probabilistic model is capable of predicting cascading failure.

A probabilistic load flow model has been designed in this paper on the basis of time instance load model and time period load model. The time instance load model (stochastic load flow) for time instant t is require to predict cascading and the time period load model over a certain period T (probabilistic load flow) is used for calculation and analysis of load duration curve, mean, variance, probability density function (PDF) and cumulative distribution function (CDF). Time instance load model of k^{th} transmission line of node i for *(x, t)* can be described by a Gaussian distribution [15]

$$\omega_i = \mu_i + \sigma_i * y_i \quad (4)$$

where, ω_i = load flow sample value of line node i
μ_i = load flow predicting value at time t
y_i = variance of load distribution of node i
σ_i = standard deviation of Gaussian distribution

Parameter σ_i describes the degree that the real load value ω_i deviates from the predictive value μ_i of node *i* at time *t* and it is used to modify the forecast load value for the node *i*. Time period load model over a period of time T of a node i described by Gaussian distribution (PDF and CDF).

$$PDF = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (5)$$

$$CDF = F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \quad (6)$$

where,
σ = Standard deviation of random variable
σ^2 = Variance of random variable
x = Random variable (% power loading in line)
μ = Mean value of random variable

The PDF can be observed as the probability that the load is larger than or equal to *x* i.e. distribution of the load. The probabilistic load model describes the probability that the load will exceed installed capacity (IC) under dynamical condition. Probabilistic modeling of power grid considers power loading of each transmission line as a random number with uniform distribution then compares this random number with forced line outage rate q (assume 80% of IC) which is used further to determine whether transmission line state is in failure or running state.

Let, C = Transmission capacity of load line
q = Probability of failure capacity (80 % of IC)
$X = (1 - q)$ = Available capacity
$p(X)$ = Probability of available capacity for x_i

$$p(X = x_i) = \{ {}^{1-q,}_{q,} \quad {}^{x_i=c}_{x_i=0} \quad (7)$$

The relative CDF is

$$p(X \geq x_i) = \begin{cases} 1, & x_i = c \\ q, & x_i = 0 \end{cases} \quad (8)$$

For cascade failure analysis it is more convenient to use outage capacity compared to available capacity. Hence, if $\bar{X}$ = outage capacity, then probability of line outage capacity of a system is:

$$p(\bar{X} \geq x_i) = \begin{cases} 0, & x_i = 0 \\ 1, & x_i = c \end{cases} \quad (9)$$

As shown in (9) probabilistic load flow model should be used as two state transmission line system to predict whether a transmission line is in running state or failure state. Based on probabilistic load flow model a statistical estimation model has been designed.

III. STATISTICAL ESTIMATION FOR PREDICTION

Assume power grid having N transmission lines with available capacity $C = (C_1, C_2, ..., C_n)$ and can be characterized as a set of mutually exclusive, discrete states $S = (S_1, S_2, ..., S_n)$ at any time T. Each state can be either the normal state or tripping state but not in both simultaneously. Thus the states are mutually exclusive and deterministic. States of power system are determined by samples from the probability distribution functions of percentage power loading in transmission line according to two point distribution functions as shown in (9). A statistical estimation, as shown in Fig.1 is used to

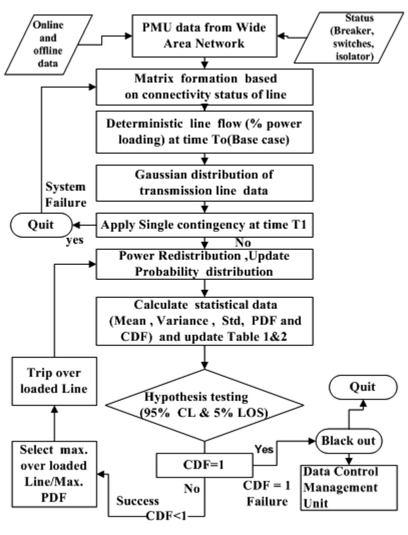

Fig. 1. Flow chart of cascading failure in power grid

test whether a transmission line is in running state or in failure state. Initially at base case, suppose there are N transmission lines in power grid with initial load capacity and flow which is Gaussian distributed from $[l_{min}, l_{max}]$ with 95% confidence level (CL) and 5% level of significance (LOS). 5% (LOS) is a chance for failure or rejection of test.

The decision rule is set according to confidence level boundaries (estimated range of values with specified probability of containing true values). Upper Bound and Lower Bound (UB, LB) for confidence intervals are computed from sampling distribution. The test rejects the estimated value when it lies outside the computed confidence interval for the parameter. If the random variable (line flow data) fitted in the Gaussian distribution (95% CL, 5% LOS) and $CDF < 1$, indicate normal state whereas $CDF = 1$ indicate blackout. For prediction of cascade link failure correlation technique is used to understand clearly the interrelationship between line loading growth patterns.

IV. PERFORMANCE EVALUATION AND SIMULATION RESULTS

As shown in Fig. 2, IEEE 30 bus test-bed system is used for deterministic load flow as well as verification of probabilistic frame work. Test

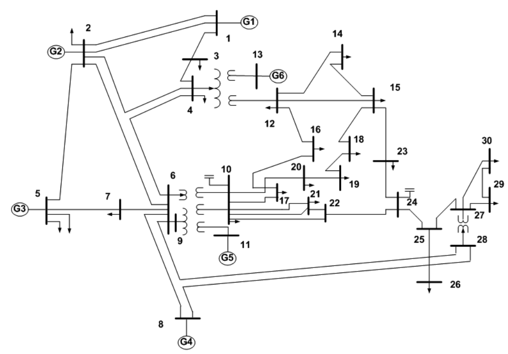

Fig. 2. IEEE 30 Bus Test system

bus system consists of 30 buses, out of which 6 buses are generator bus and 23 are load buses with 42 transmission lines consisting of 289.1 MW generation and 283.4 MW load flow capacity of transmission line (standard data).

A. Graphical Model of the Grid

The topology of the power grid network can be represented graphically by allocating generator and load buses as a node (V, vertices) and transmission lines, transformers as a link (E, Edges). Graphical model (G) is represented as

$$G = (V, E, W) \quad (10)$$

Weighted directed tree $T = (V, E, W)$ of the bus along with maximum capacity link connectivity (red color link) is shown in Fig. 3 where W is the weight function (line admittance) for the edges (transmission lines). The number across transmission line representing respective line numbers and values inside the bracket indicating respective line admittance values. Minimum admittance line carries maximum power flow. One of the algorithms for calculation of connectivity and maximum power flow in the transmission lines is the Minimum Spanning Tree (MST) algorithm which presents local to global connectivity and feasible solution of power flow in the grid. MST of test model using Kruskals method is shown in Fig.3 where red lines showing minimum admitance or maximum load bearing line starting from slack bus (node 1) to all buses .As MST contains all $(n-1)$ nodes so there is a possibility or existence of cascade failure. Spanning tree model also used in [2], [3], [4], [5], [6] for optimal PMU placement. The PMU placement on the buses for the IEEE 30 bus system as per [16] are bus number 2,4,6,9,12,10,15,25, and 27. Performance evaluation of the test system for this paper based on the normal operating condition and under consideration of line tripping.

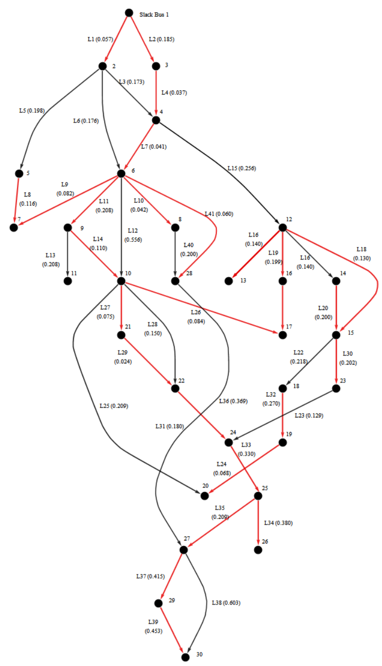

Fig. 3. Minimum spanning tree of IEEE 30 Bus System

B. Performance evaluation of load flow

The base case (Case-1) as shown in Fig. 4, is a graph of percentage power loading in lines as a function of line number under normal working condition. Fig. 5 is a Gaussian distribution plot where as in Fig. 6 CDF displays a cumulative probability plot of the load flow data. As shown in Fig. 5 PDF curve is Gaussian and in Fig.6 all the data is properly fitting inside the confidence level hence all lines are healthy at the base case. Assume the transmission line capacity or force outage rate (FOR) is 80% of IC for analysis of load flow under consideration of line tripping. Cascading is initiated by exceeding line load on selected line (for example highly loaded line) and if load exceeds to FOR, transmission line will trip. Probability of next tripping line is calculated using statistical decision

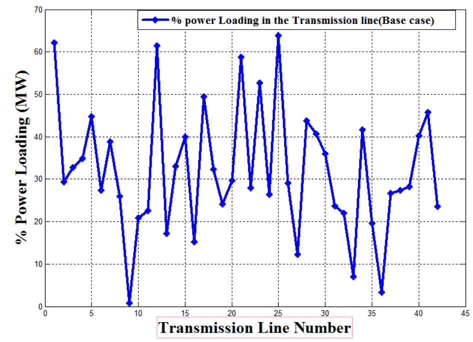

Fig. 4. Case1- Power loading as a function of line number

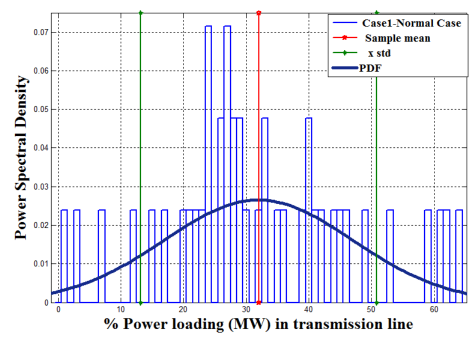

Fig. 5. Case1- PDF as a function of Transmission line loading

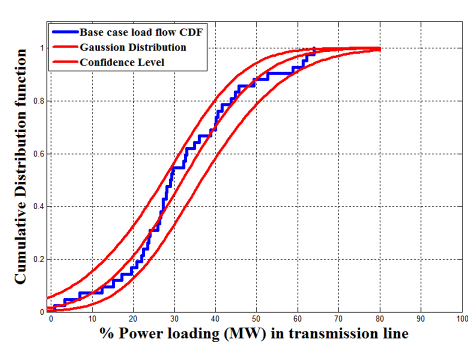

Fig. 6. Case1-CDF as a function of line loading

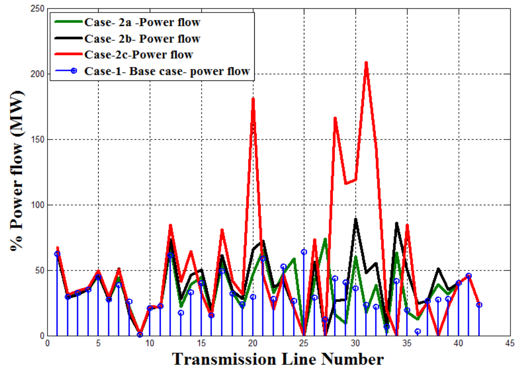

Fig. 7. power loading as a function of line no. in Case2

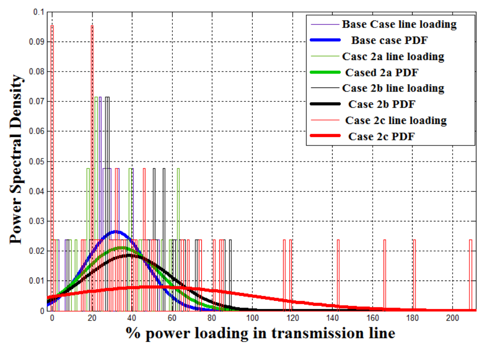

Fig. 8. PDF plots of line data with sample mean and variance.

theory (Hypothesis testing). Performance evaluation of test bed system under dynamic condition is initiated by tripping of line L25 (between node 10 to 20), as shown in Figs. 2 and 3. Tripping of L25 redistributed the power flow in the grid. Redistributed flow and corresponding deterministic data are shown in Fig.7 and Table II for Case 2a. Accordingly new probability distribution with sample mean and zero mean are shown in Figs.8 and 9 (Case 2a). New statistical data (estimated mean, variance, CDF and estimated co- variance)

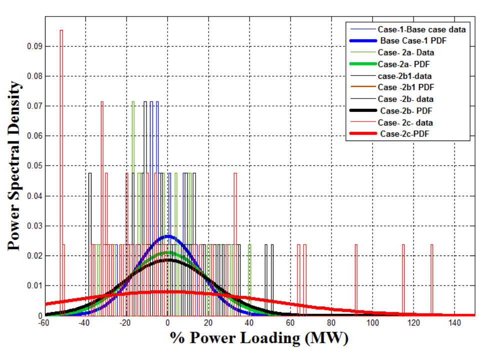

Fig. 9. PDF plots of line data with zero mean and variance.

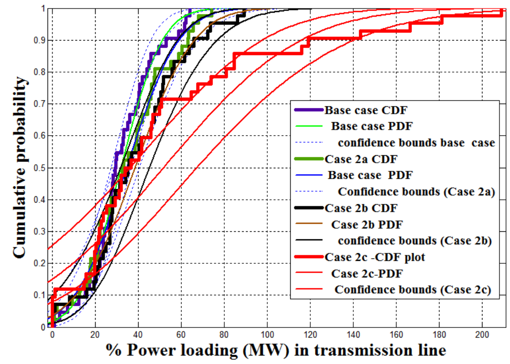

Fig. 10. CDF plots of transmission line data

TABLE I
PROBABILISTIC LOAD FLOW ANALYSIS

Case	Tripped line	Mean (μ), Standard deviation (σ)	CDF, line loading	LB / UB	Estimated covariance	Line tripping probability
Case-1	Base Case	μ=31.97 σ=15.05	0.9834 at 64%	0.9424 /0.9964	μ=5.39 σ=2.80	all lines are healthy
Case-2a	L25	μ=34.63 σ=18.95	0.9855 at 75%	0.9474 /0.9970	μ=8.55 σ=4.44	L27
Case-2b	L27	μ=38.47 σ=21.53	0.9906 at 87%	0.97 /0.9984	μ=11.03 σ=5.72	L38
Case-2c	L38	μ=51.71 σ=49.95	0.9999 at 210%	0.9921 /0.9992	μ=59.46 σ=30.83	Black-out

from Figs. 8,9, and 10 for case 2a are updated in Table I. According to hypothesis testing probability of next tripping line is L27.

Tripping of L27 (between node 10 to 21) redistribute power flow, accordingly deterministic as well as probabilistic data changed which is reflected in Table 1 and II. Deterministic load flow in case 2b (tripping of L27) is shown in Fig. 7. Fig 8 is a Gaussian distribution (Case 2b) about its sample Mean and increased variance whereas zero mean, variance for the case 2b is plotted in Fig.9. Corresponding CDF with 95% confidence bond and 5% level of significance (Case 2b) is shown in Fig. 10. As CDF plot is fitted under LB and UB with the CDF value less than 1, test shows success and declares probability of next tripping line is L38 (highest CDF).

Case-2c in Figs. 7,8,9, and 10 is a contingency

TABLE II
DETERMINISTIC LOAD FLOW ANALYSIS

Case	Tripp-ed Line	Highly loaded Line	FOR (q)	Available Capac-ity (1-q)	Line Trip-ping Proba-bility.
Case-1	Base Case	Under IC	Under FOR	Full	All lines are healthy
Case-2a	L25	L27(74.2%)	1	Nil	L27
Case-2b	L27	L38(86.1%)	1	Nil	L38
Case-2c	L38	–	–	–	Blackout

analysis after the tripping of L38 for the system. Table I shows simulation results based on statistical decision theory and Table II results are based on deterministic load flow contingency analysis.

Comparison of Table 1 and 2 concludes that with deterministic load flow analysis, after tripping of L38, system leads to blackout (Case-2c in Fig 7) where as entries in Table 1 and PDF plots in Figs. 8 and 9 clearly indicate Gaussian to non Gaussian distribution of load flow in transmission line. CDF plots in Fig. 10 indicating effect of cascade failure which lead to system blackout after tripping of L38. Hence, proposed probabilistic framework identified critical links in cascade failure.

V. CONCLUSIONS

The probabilistic framework for smart grid cascade failure analysis has been explained in this paper along with cascading prediction based on statistical decision theory. Simulation result shows that a probabilistic distribution of blackout size has heavy tails where distribution had been no more Gaussian. Hence, probabilistic load flows are very much useful for the analysis of variation in nodal generation and loads over a period of time. load flow analysis concluded that, one statistical load flow could replace large number of deterministic load flows to cover every possible contingency in the system. Next generation smart grid demands real time multiple contingency analysis with self-healing and robust technology. This leads to various open research areas in the field of smart grid resilience of cascade failure. Some of the issues are prediction as well as prevention of cascade failure with real time data and multiple line tripping analysis.

VI. ACKNOWLEDGEMENT

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